A novel model-based fault detection method for temperature sensor using fractal correlation dimension

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**ABSTRACT**

The direct residual-based fault detection method compares the difference between measured and estimated data of a process variable. Its correct fault detection rate is low due to the noise in measured signals. A novel method using fractal correlation dimension (FCD) is developed, in which FCD deviation is adopted instead of direct residual. The method is validated by detecting fixed and drifting bias faults generated in supply air temperature sensor of air handling unit (AHU) system. The setting of three main parameters including embedding dimension, time delay parameter and length scale, is investigated to find out the influence on calculating FCD values. The results show that it is more efficient to detect relatively small bias fault under noise conditions, although it needs a period of time to collect measured data. As a promising and practical tool, a hybrid fault detection technique combining FCD with direct residual should be conducted in further investigation to identify the generated fault under inevitable noise conditions.

**1. Introduction**

As the key components in heating, ventilation and air conditioning (HVAC) systems, sensors provide the basic information for operation and control system. It is very important to maintain an accurate and reliable operation without any faults in sensors. Unfortunately, sensor fault may occur in sensing device or its electronic components after long-term operation [1]. The common sensor faults can be classified as four different categories including fixed bias, drifting bias, precision degradation and complete failure [2]. All of them may decrease the control efficiency of controller, and result in poor operation or the invalidation of advanced optimal strategies.

Dozens of different methods have been applied to detect faults and their major differences are the knowledge used for formulating the diagnostics. The state-of-the-art fault detection and diagnosis (FDD) methods can be divided into four categories, including redundancy related [3], quantitative model-based [4–6], qualitative model-based and process history based [7] methods. Each method possesses some strengths as well as some weaknesses and suitability [8], and none is generic and perfect for any desirable cases. Only a few have actually been employed in FDD implementations although a number of physical models were developed for HVAC systems over the decades.

According to the engineering literatures, some of the FDD methods directly compare the residuals or deviations between measured and estimated values, and these tools can be defined as direct residual-based methods. For the widespread use of these methods, a crucial step is the ability to develop an accurate reference model of the equipment or system characterizing its fault-free operation [9–12]. Then, such a model can detect faulty operation by tracking the residuals or deviations between measured and estimated performance, and by identifying fault occurrences when these deviations exceed pre-selected thresholds.

There are some reference models to acquire the accurate fault-free parameters. Using the first law of thermodynamics with steady-state mass or energy conservation, a robust fault diagnosis method [13] can examine and minimize the sum of squared deviations over a period. This method is validated to evaluate soft sensor faults (biases) for temperature sensors and flow meters in central chilling plant. Other mathematical models including black-box multivariate polynomial methods, specifically radial basis function and multilayer perceptron, the generic physical component model [11,12], artificial neural network [14], rough set approach [15], transient pattern analysis [16] and others, are used to get deviations for well suited automated FDD in HVAC equipments and systems.

Residuals or deviations are always obtained by data processing of measured variable [17] about the actual process. This processing
The direct residual-based methods, however, have some weaknesses in fault detection. In practice, some inevitable noise may decrease the correct fault detection rates. Further, under noise conditions, the specified thresholds may impact on faulty or fault-free decision so significantly that it is very difficult to set a perfect value. If the threshold is far beyond noising ranges, the methods could not identify fault symptoms or state fluctuations especially when the faults present unobvious effects on the measured parameters. As to a small threshold, however, some influences, such as the error of data filtering, the disturbance of external environment and the time delay from measuring equipments, will be impossible to prevent. In this case, the methods may call good processes faulty and lead to large false alarm rates.

The fault detection method using fractal correlation dimension (FCD) algorithm [19] may cover the deficiency of direct residual-based method. FCD measures the probability of two points chosen at random. Within a certain distance between these two points, how this probability changes is examined with the distance increasing. As a non-linear signal processing tool, the FCD algorithm employs a dimension value to represent the curve variation of the original time series. The FCD value contains the characteristic information which includes the important fractal theories such as the self-similar characteristic and fractal dimension, and the curve variation refers to the time-dependent variation in the parametric curves. FCD could reflect the irregular or non-stationary nature of signals, and even the operational state of equipment or system. To extract the FCD of feature vector will be beneficial to detect or identify the faulty characteristic signals.

In this study, the principle of FCD is formulated and a novel FCD-based method is developed to detect fault generated in AHU supply air temperature sensor. The multiple-point fast Fourier transformation (FFT) algorithm is used to filter noise and the novel method is validated by detecting fixed and drifting bias faults. Besides, three main parameters are investigated to get the desirable FCD values.

2. Principle of FCD

In fractal geometry, the fractal dimension is a statistical quantity to indicate how completely a fractal appears to fill space. There are many specific definitions of fractal dimension. Partly due to the ease of implementation, FCD algorithm is widely used in practice. FCD algorithm is a measure of the dimensionality of the space occupied by a set of random points.

Usually, the practical measured data are irregular signals to display fractal characteristics at certain scales. As one of the achievements of chaos theory, an attractor is a set towards which a dynamical system evolves over time. The points can get close enough to the attractor, and an attractor can be a point, a curve, or even a complicated set with a fractal structure. If the system operation deviates from the fault-free state, the corresponding attractors, as well as the consequent FCD, will change. To put it in another word, the changes of system state will induce different FCD, which implies the changing state for system. Thus, fractal characteristics can use an FCD value to depict structural characteristics from irregular signals.

By using Grassberger and Procaccia (GP) algorithm [20], the phase space reconstruction can be available from the observed one-dimensional time series, and the FCD value representing dynamical system attractor can be calculated from the observations of phase time series. The curves for ln(C(r)) versus ln r can be plotted based on GP algorithm. By regulating the values of m and τ until the slope of the curve’s linear part is almost invariable, the slope of this linear part is calculated by a least-square fit.

The time series can be described as

\[ \{X_i\}, i = 1, 2, \cdots, n \]

where \( n \) is the length of time series.

The idea of chaotic dynamics reconstruction technique stems from the embedding theorem which regards a one-dimensional chaotic time series as the compressed information of a high dimension space [21]. \( m \) denotes the embedding dimension, and there are \( m \) points of data in each phase space. Then, the phase of spatial data can be represented as a series of points in an \( m \)-dimensional space and the \( j \)th scalar time series can be recorded as

\[ x_j(m, \tau) = (X_{1}, X_{1+r}, \cdots, X_{1+(m-1)\tau})^T, \quad j = 1, 2, \cdots, n - (m - 1)\tau \]

where \( \tau \) is the time delay parameter and \( \tau = k\Delta t \).

According to the method mentioned above, the time series with \( n \) points of data is divided into \( n_m \) groups.

\[ n_m = n - (m - 1)\tau \]

where \( n_m \) is the number of the points or coordinate vectors in the fractal set.

The \( m \)-dimensional hypersphere radius, is represented by Euclidean distance.

\[ r_{ij}(m, \tau) = \|x_j(m, \tau) - x_i(m, \tau)\|^2 = \sum_{k=0}^{m-1} (x_{i,k\tau} - x_{j,k\tau})^2 \]

The center of a hypersphere can be defined as and the radius as. For each \( i \) with fixed value, a radius can be calculated from the
phase space distances by the spherical triangle method [22], and
then the radiuses from different $i$ can be obtained. Changing the
center of the reproduced hypersphere will get a series of small
spheres. If $r$ is defined as a length scale, the radius smaller than the
scale will be reserved but others will be discarded. Thus, the ratio
between the number of small spheres and the number of total
spheres is defined as the correlation integral function $C(r)$.

$$C(r) = \frac{1}{nm(nm - 1)} \sum_{i=1}^{nm} \sum_{j=1}^{nm} H[r - r_i(m, \tau)], i \neq j$$  \hspace{1cm} (5)

$$H[r - r_i(m, \tau)] = \begin{cases} 1 & r - r_i(m, \tau) \geq 0 \\ 0 & r - r_i(m, \tau) < 0 \end{cases}$$  \hspace{1cm} (6)

$$r = (r_{ij,\text{max}} - r_{ij,\text{min}}) \frac{i + 1}{p + 1}, i = 1, 2, \ldots, p$$  \hspace{1cm} (7)

Where $H$ is the Heaviside step function.

In view of $r_i(m, \tau) = r_i(m, \tau)$, using the following formula can
avoid the same calculation and save a half of computation.

$$C(r) = \frac{2}{nm(nm - 1)} \sum_{i=1}^{nm} \sum_{j=1}^{nm} H[r - r_i(m, \tau)]$$  \hspace{1cm} (8)

$$C(r) = \lim_{n_m \to -\infty} \frac{1}{n_m^2} \sum_{i=1}^{n_m} \sum_{j=1}^{n_m} H[r - r_i(m, \tau)]$$  \hspace{1cm} (9)

Because $r$ is sufficiently small and the number of observed values $n_m$ is large enough, the reconstructed phase space attractor of
FCD can be derived as

$$D_c = \lim_{r \to 0} \frac{d[\ln C(r)]}{d[\ln r]}$$  \hspace{1cm} (10)

or

$$D_c = \lim_{r \to 0} \frac{\ln C(r)}{\ln r}$$  \hspace{1cm} (11)

Obviously, $C(r)$ is proportional to the number of point pairs.
These point pairs are separated by some distances less than $r$ in the
fractal set. If the point system is examined as a fractal set, the graph of $C(r)$ in logarithmic coordinates must be a linear function with the
slope equaling to the FCD of system.

3. Data acquisition and load conditions

The simulated data from the TRNSYS simulation platform [23]
are used to validate the sensor FDD method. Besides, an AHU
process model is developed to provide the fault-free variables. The
novel fault detection method is validated under the same load
conditions.

3.1. AHU system

The AHU system with a control system for adjusting supply air
temperature [2,23,24] is installed in an office building. Fig. 1 illustrates
the schematic diagram for AHU system including two mass
circulations (chilled water and supply air circulation) and one
control loop for supply air temperature.

AHU waterside includes primary and secondary chilled water
loops. In airside, a fixed amount of outdoor air mixes with recirculated air from air-conditioned spaces. The mixed air is cooled
down by the chilled water inside AHU coil. As shown in Fig. 1, the
control loop includes three main components involving a tempera-
sensor, a PID controller and a valve actuator. To maintain the
supply air temperature at the pre-selected set point, a PID
controller regulates the water valve to provide sufficient chilled-
water flow rates.

3.2. Simulation platform

A simulation program developed on TRNSYS Simulation Program [23] is applied as the validation platform. The simulation includes
the program of waterside and airside systems. The set
point of supply air temperature is 14 °C.

To simulate the fault generated in temperature sensor, a fault
generator by using some equations is developed in TRNSYS to
introduce a required fault signal at a certain moment. Two types of
faults, fixed bias and drifting bias, are validated in supply air
temperature sensor.

A noise generator by using some equations is developed in
TRNSYS program to introduce a stochastic noise in the range from
$-0.3$ °C to $+0.3$ °C. These measured data under noise conditions are
recorded to calculate their FCD values.

The electric noise is a random fluctuation in sensor signals. It can
be produced by several effects such as thermal noise, shot noise and
other noise depending on manufacturing quality or external envi-
ronment. In control systems, the noise is an error or undesired
random disturbance in useful signals. The measurement accuracy
refers to how close the measured value is to the true or accepted
value. The presence of noise will decrease the measured accuracy and thus leads to imprecise sensor readings. Similar to most fault
detection methods, the novel method will make a wrong decision
due to the noise in measurement or the low measurement accuracy.

3.3. AHU process model

The AHU coil process model, which has been illustrated in
several published works [19], is set up to get the fault-free or
normal behavior/reference. As a classical quantitative model-based
method, this model constructs the mathematical relationships
between inputs and outputs, based on the principle of mass and
energy conservation. Fig. 2 illustrates the schematic diagram for
AHU coil process model. Five inputs are directly or indirectly
derived from the normal sensors installed in AHU systems. The
inlet airflow rate is obtained from the psychrometric chart on
the basis of inlet air dry-bulb temperature and wet-bulb tempera-
ture. In Fig. 2, $t_{in,a}$ is inlet air temperature, $\epsilon_{in,a}$ is inlet air humidity
ratio, $C_{in,a}$ is position signal for inlet air damper, $t_{in,w}$ is inlet water
temperature, $C_w$ is position signal for water valve, $t_{sup,a}$ is supply air
temperature, $t_{out,a}$ is outlet air humidity ratio, and $t_{out,w}$ is outlet
water temperature.
3.4. Load conditions

Different loads and operation conditions may challenge the robustness of the fault detection strategy. In this study, three typical summer days are selected as the load conditions and entitled Load 1, Load 2 and Load 3, respectively. These loads have the same internal loads but different weather conditions. For each load, the time-dependent variations of three parameters including outdoor air dry-bulb temperature, outdoor air humidity ratio and AHU cooling load are shown in Fig. 3.

4. Outline of the novel fault detection method

Statistically, the direct residual-based method always uses considerable residuals between measured and estimated data to determine whether fault generates or not [25, 26]. To avoid directly comparing the numerous data under noise conditions, a novel method employs FCD deviation instead of direct residual. The logic diagram of this novel fault detection method based on FCD is shown in Fig. 4. The multiple-point FFT filter [27] is employed to pre-process the measured data to significantly reduce the impact of noise, and then the measured FCD value, $D_{\text{meas}}$, can be calculated by the filtered measured data from TRNSYS simulation platform. Simultaneously, the estimated data are obtained from AHU coil process model, and then the estimated FCD value, $D_{\text{est}}$, can be calculated. The FCD deviation, $\Delta D_C$, is used to identify faults by comparing with the pre-selected threshold.

$$\Delta D_C = D_{\text{meas}} - D_{\text{est}}$$ (12)

The number of point in FFT filter is determined by that, under fault-free conditions, whether the measured FCD value is equal to or very close to the estimated one. If two FCD values are very close or their difference is lower than the threshold, the selected number of point is considered to be proper. When the noise is in the range from $-0.3 \, ^\circ \text{C}$ to $+0.3 \, ^\circ \text{C}$, it is found that 10 point is the proper one.

Similar to the selection of the number of point in FFT filter, whether the noise should be filtered out or not is also determined by whether the two FCD calculations are very close to each other under fault-free conditions. If the measured FCD is equal to or very close to the estimated FCD, the noise is not filtered out. If the difference between two FCD values is large enough to exceed the threshold, the noise must be filtered out.

5. Simulation results

On the simulation platform mentioned in Section 3, the novel method is validated in three typical summer days. In this section, only the simulation results under Load 1 are discussed to illustrate the temperature variation and the FCD calculations. Fig. 5 shows the flow chart of simulation and validation for novel FCD-based fault detection method. Fault-free signals of supply air temperature are estimated from the AHU coil process model. Meanwhile, the measured data under a noise with $0.3 \, ^\circ \text{C}$ maximum amplitude are recorded at 10 s intervals. In the occupied period of time from 09:00 to 16:00, 2520 points in total are thus obtained. To reduce the influences of noise and the error from measuring equipments, 10-point FFT is adopted as the data filtering tool to process the measured data. Two types of faults, fixed bias and drifting bias, are used to validate the novel FCD-based method. For FCD calculation, 37 and 10 are respectively selected as the
embedding dimension $m$ and the time delay parameter $\tau$. Also, the pairs of $\ln r - \ln C(r)$ data are selected in suitable scaling regime. Figs. 6–8 depict the time-dependent plot variation and the FCD calculation of fault-free, fixed bias and drifting bias. Both measured and estimated temperature curves depict the time-dependent variations. The measured curve consists of the filtered temperature measurements which are processed by 10-point FFT filter. The estimated curve consists of the estimated temperature obtained from the AHU process model. The FCD calculations for both curves are marked in the top of the figures. The measured FCD value, $D_{C,\text{meas}}$, denotes an FCD value calculated by 10-point-FFT measured data, and the estimated FCD value, $D_{C,\text{est}}$, means an FCD value calculated by estimated data. Six fixed-bias faults including $0.2\degree C$, $0.3\degree C$ and $0.5\degree C$, and six drifting-bias faults including $0.1\degree C/h$, $0.15\degree C/h$ and $0.2\degree C/h$, are all introduced at 10:00.

5.1. Fault-free

Due to some noise in measuring process, the measured data from temperature sensor exhibit large fluctuations as well as high FCD value. If the fault-free measured signals from temperature sensor are filtered by 10-point FFT, the calculated FCD value is very close to that from estimated data. As shown in Fig. 6, the FCD value for 10-point-FFT measured data is 4.87, while that for estimated data is 4.63. Thus, the fault-free $\Delta D_C$ is 0.24.

5.2. Fixed-bias fault

As illustrated in Fig. 7, various fixed-bias faults show similar curve variations of measured data. A sensor fault produces a faulty reading, and this reading will be input into controller. To ensure the supply air temperature at the set point, the controller should regulate the water valve to change the water flow rates into AHU coil. Thus, the actual supply air temperature changes and the sensor reading will approach the set point. The difference between actual temperature and sensor reading may be approximate to the fault bias. The measured temperature curves are slightly different variations due to changed water flow rates and keep more similar due to fluctuation near the set point.

Negative bias fault presents lower measured value than the estimated one, but positive bias fault presents higher measured value than the estimated one. In other words, at a fixed set point, a negative temperature sensor bias will lead to a higher supply air temperature, and a positive bias will produce a cooler supply air temperature [24]. Following the appearance of positive or negative fixed-bias fault, both measured and estimated temperature curves present an immediate striking step and then remain similar but deviated. Obviously, the higher fixed bias is, the larger curve deviations present.

Greater variations in temperature curves will result in larger changes in FCD value. As depicted in Fig. 7, the FCD values for measured data with fault are all beyond 3.29 and greater than that for fault-free estimated data. The fault-free estimated FCD values are all less than 2.85 and show a declining trend with the fault bias increasing. For $0.5\degree C$ fixed-bias faults, the estimated FCD values are only up to 1.0 and much less than the measured ones. Generally, the higher fixed bias will result in larger FCD deviation.

5.3. Drifting-bias fault

As depicted in Fig. 8, the drift-bias fault will result in gradual drifting sensor readings as time goes on. Different from fixed-bias fault, the temperature curve will not always be accompanied with
a striking step when fault occurs abruptly, but there still exist frequent fluctuations which can be represented by FCD.

Six drifting-bias faults exhibit very similar and no significant change in curve variations of measured temperature. The FCD values for negative drifting faults are all higher than 3.6 and very close to each other, but the FCD values for positive drifting faults are all greater than 4.1. The FCD values for estimated temperature curves show a decreasing trend with the increasing of drifting bias. The FCD value is 0.92 for $-0.1 \degree C/h$ drift, 0.82 for $-0.15 \degree C/h$ drift and 0.72 for $-0.2 \degree C/h$ drift. Also, the FCD value is 1.56 for $0.1 \degree C/h$ drift, 1.16 for $0.15 \degree C/h$ drift and 1.02 for $0.2 \degree C/h$ drift.

With the increasing of drifting bias, the estimated temperature curve gradually deviates from the measured one. Further, the deviations between two curves grow larger and larger as time goes on. This is the same to the corresponding FCD. Larger curve deviations will lead to greater FCD variations which represent more distinct faulty characteristics. The novel FCD-based method, therefore, could be capable of identifying the occurred fault in a valid and reliable way.

6. Evaluation of the novel method

In this section, the novel fault detection method is evaluated by compared with the direct residual-based method. Three loads including Load 1, Load 2 and Load 3 can provide different external loads and operation conditions. Without consideration of the time delay for measuring instruments, a noise ranging from $-0.3 \degree C$ to $+0.3 \degree C$ is introduced in sensor outputs as the noised temperature measurement.
6.1. Direct residual-based method

The method directly compares the residual between measured and estimated temperature. Two parameters, the correct fault-free detection rate and the correct fault detection rate, are used to represent the performance of direct residual-based method. The correct fault-free detection rate denotes the probability of calling a good process good, and the correct fault detection rate denotes the probability of calling a faulty process faulty.

The correct fault-free detection rates of direct residual-based method are listed in Table 1. If the temperature threshold is set at 0.2 °C, all correct fault-free detection rates are lower than 73%. For the temperature threshold setting at 0.3 °C, all correct fault-free detection rate is greater than 84% but lower than 92%.

The correct fault detection rates of direct residual-based method are listed in Table 2. The correct detection rates of negative bias are larger than that of positive. This is mainly because the AHU system is more sensitive to negative fault than to positive, and the negative fault is therefore convenient to be detected. Further, if the fixed or drifting bias is small, the correct fault detection rate is also small. If the temperature threshold is set at 0.2 °C, three fixed-bias faults including −0.5 °C, −0.3 °C and +0.5 °C, and three drifting-bias faults including −0.2 °C/h, −0.1 °C/h and +0.2 °C/h, have the correct fault detection rates greater than 72%. If the threshold setting at 0.3 °C, only two fixed-bias faults including −0.5 °C and +0.5 °C, and

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Load 1</th>
<th>Load 2</th>
<th>Load 3</th>
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<tbody>
<tr>
<td>0.2 °C</td>
<td>64.3</td>
<td>72.6</td>
<td>61.5</td>
</tr>
<tr>
<td>0.3 °C</td>
<td>91.0</td>
<td>88.9</td>
<td>84.3</td>
</tr>
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Fig. 8. Time-dependent variation of temperature plot for drifting-bias fault under Load 1.
two drifting-bias faults including $-0.2 \, ^\circ\text{C/h}$ and $+0.2 \, ^\circ\text{C/h}$, have the correct fault detection rates greater than 68%.

The direct residual-based method may be more suitable for detecting the relatively large bias fault under noise conditions. To increase the correct fault-free detection rate, large threshold such as $0.3 \, ^\circ\text{C}$ should be set, but this will lead to low correct fault detection rates.

### 6.2. Strengths of the novel method

By comparing with the direct residual-based method, the FCD-based method can considerably reduce the number of deviations between measured and estimated data. In view of evident non-linear characteristic for air-conditioning system, the deviations between measured and estimated data are always irregular either for fixed-bias or for drifting-bias fault. Within a certain period of time, an FCD value can represent the variation of correlation feature in temperature curves. Further, the FCD is very sensitive to this variation. In this way, the novel FCD-based fault detection method can detect the generated bias fault.

On the other hand, the FCD-based method can significantly increase the correct fault detection rates. As summarized in Table 3, either negative or positive faults show higher FCD deviations than fault-free. The novel FCD-based method can characterize the fault curve variations and distinguish their symptoms. If the dimension threshold is set at 0.9, the threshold is so far beyond the FCD deviations of fault-free behavior that the correct fault-free detection rate and the correct fault detection rates are all 100%. If the dimension threshold is set at 1.2, the correct fault-free detection rate is still 100% and the correct fault detection rate is 97.2%. The novel FCD-based method, therefore, is superior to the direct residual-based method in reducing the impact of noise.

It is very important to select a proper dimension threshold. The threshold setting at 0.9 can detect all faults, but the threshold setting at 1.2 can not detect $+0.2 \, ^\circ\text{C}$ fixed-bias fault under Load 2. A desirable threshold should be not only larger than the FCD deviations of fault-free behavior, but also lower than the FCD deviations of fault behavior. Further, due to the complexity of dynamic non-linear system, the varying operational conditions will lead to distinct changes in the AHU parameters when the same fault generates. The resulting FCD calculations are also dissimilar, so it is very difficult to find only one threshold to suit all AHU systems.

### 6.3. Weaknesses of the novel method

The typical weakness of novel FCD-based method is that a highly accurate model is a prerequisite for fault detection especially in the field of air-conditioning system with strong non-linearity. This is the same to other model-based methods. The operating parameters will be changed dramatically resulting from the frequent changes in outdoor meteorological conditions, indoor occupants, or even certain parametric settings. An exact physical/mathematical model or an accurate prediction tool, therefore, is essential to get available operating data under fault-free conditions.

To use the proposed method in a real system, quantitative accuracy requirement is needed if the model accuracy affects the fault detection capability significantly. The acceptable model accuracy should ensure that the fault-free estimated value must be very close to the fault-free measured one. Thus, the difference between measured and estimated under fault-free conditions is

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<table>
<thead>
<tr>
<th>Table 2</th>
<th>Correct fault detection rates of direct residual-based method [%].</th>
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<tbody>
<tr>
<td>Threshold</td>
<td>Load 1</td>
</tr>
<tr>
<td></td>
<td>$-0.5 , ^\circ\text{C}$</td>
</tr>
<tr>
<td></td>
<td>Fault-free</td>
</tr>
<tr>
<td>0.2 , ^\circ\text{C}</td>
<td>99.5</td>
</tr>
<tr>
<td>0.3 , ^\circ\text{C}</td>
<td>90.2</td>
</tr>
<tr>
<td>0.2 , ^\circ\text{C}</td>
<td>99.7</td>
</tr>
<tr>
<td>0.3 , ^\circ\text{C}</td>
<td>93.1</td>
</tr>
<tr>
<td>0.2 , ^\circ\text{C}</td>
<td>99.8</td>
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<tr>
<td>0.3 , ^\circ\text{C}</td>
<td>96.4</td>
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<th>Table 3</th>
<th>FCD calculations of the novel method [\ldots].</th>
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<tr>
<td>Fault-free</td>
<td>Load 1</td>
</tr>
<tr>
<td>$D_{c,\text{mean}}$</td>
<td>4.87</td>
</tr>
<tr>
<td>$D_{c,\text{est}}$</td>
<td>4.01</td>
</tr>
<tr>
<td>$\Delta D_c$</td>
<td>0.24</td>
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<th>Drifting bias</th>
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<tr>
<td>$-0.5 , ^\circ\text{C}$</td>
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<tr>
<td>$-0.1 , ^\circ\text{C/h}$</td>
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<tr>
<td>$-0.15 , ^\circ\text{C/h}$</td>
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<tr>
<td>$-0.2 , ^\circ\text{C/h}$</td>
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lower than the threshold. Only in this case, the proposed method can be efficient to make the correct decision.

On the other hand, the novel FCD-based method needs a period of time to detect a bias fault. This is the undesirable weakness for this novel method. Fortunately, the direct residual-based method can quickly detect the large bias fault under noise conditions. A hybrid fault detection technique combining these two methods, therefore, can use direct residual-based method to detect the large bias fault, and use FCD method to discover the relatively small bias fault.

7. Parametric settings for FCD calculation

Selecting an appropriate scaling regime or straight line portion from logarithmic plots for many chaotic systems is significant to the accurate calculation of FCD by using GP algorithm. When applying FCD to depict the system behavior, three parameters including embedding dimension $m$, time delay parameter $\tau$ and length scale $r$, should be selected optimally for a specified dynamic system. In order to obtain the optimum, the impacts of these main parameters on FCD calculation are investigated in this section. The estimated data from AHU coil process model are selected from 09:00 to 16:00.

7.1. Embedding dimension $m$

Theoretically, FCD denotes the geometric properties of attractors. If $m$ is too small, the attractor may be so folded that self-intersection will appear in some places. In this case, a small neighborhood in these intersection regions may contain much more additional points from different parts of attractor. If $m$ is too large, this may be theoretically possible but unsuitable in practical applications. The increase of $m$ would greatly increase large amount of FCD calculation, and the impact of noise and rounding error will be strongly increased as well. The geometric structure will be completely open if $m$ is greater than the minimum embedding dimension; hence the calculations are independent with $m$, and the FCD is selected when the geometric calculations stop varying.

An approach with much costly calculation but more reliable algorithm gradually increases embedding dimension until the FCD value will never change, thus the current value shall be the smallest embedding dimension. Fig. 9 depicts the variation of $\ln C(r)$ versus $\ln r$ with a constant $\tau$ of 10 and $m$ value from 10 to 100. The curve slope for $\ln C(r)$ versus $\ln r$ enlarges with the increasing of $m$. When $m$ is greater than 30, the slope presents less variation. Besides, $m$ value in the range from 30 to 40 has no distinct influence on the curve slope. The integer 37, which is big enough to satisfy the character for reconstructed vector space, is selected as the $m$ value to reconstruct the image of original dynamical system.

7.2. Time delay parameter $\tau$

The effect of $\tau$ on the quality of FCD reconstruction is more important than that of embedding dimension $m$ [28]. To guarantee a reliable estimate of FCD, the delay time should be substantially reduced when the embedding dimension increases. Fig. 10 illustrates the variation of $\ln C(r)$ versus $\ln r$ with a constant $m$ of 37 and $\tau$ from 5 to 50. If $\tau$ value is in the region from 5 to 10, the reconstructed image exhibits preferable linearity and the slope of $\ln \tau - \ln C(r)$ linear curve has no significant changes. This region can be considered as the obvious fractal scaling regime. In the study, the integer 10 is recommended for $\tau$ to obtain a sound FCD estimation.

7.3. Length scale $r$

The $\ln r - \ln C(r)$ plot usually presents some irregularities for small values of $r$ because a finite amount of data is involved in practice, and generally saturates for large values of $r$ because the whole data set may be enclosed in large partition [29]. In the bounds of the

![Fig. 9. Variation of $\ln C(r)$ versus $\ln r$ with $\tau = 10$ and $m$ value from 10 to 100.](image)

![Fig. 10. Variation of $\ln C(r)$ versus $\ln r$ with $m = 37$ and $\tau$ value from 5 to 50.](image)

![Fig. 11. Variation of $\ln C(r)$ versus $\ln r$ for $m = 37$ and $\tau = 10$.](image)
minimum and maximum phase-space distances, 20 values are selected equidistantly as the length scale r. Under a specific AHU load, Fig. 11 illustrates the ln \( r \)–lnC(r) plot for six different fixed-bias and fault-free measured temperatures. The plot slope in the range of recognized scaling regime is defined as the FCD of attractor to represent curve reconstruction. It can be seen that there are various scaling regimes for different curve reconstruction. Thus, the corresponding FCD values are various for different faulty and fault-free behaviors. In order to eliminate the impact of a few non-linear points, the curve slope should be selected in an appropriate recognized scaling regime. The slope for each reconstruction is not always with clear linearity and the related slope may be imprecise to represent the reconstruction character. FCD values, however, may be indicative of the curve variations to some extent, especially for only purpose in comparing the difference between faulty and fault-free.

8. Conclusion

Two remarkable strengths of the novel FCD-based method are: (1) the number of residuals for comparing will be significantly reduced due to only a single numeric value utilized to characterize the curve variations; (2) compared with the direct residual-based method, the novel FCD-based method is more efficient to identify whether fault generates or not, especially for the relatively small bias fault under noise conditions. Since an FCD value is very sensible to curve variation, the novel FCD-based method is usually with high FCD deviations.

The typical weakness of novel FCD-based method is that the validity of fault-free or faulty decision is greatly dependent on a highly accurate model. Under fault-free conditions, the acceptable quantitative model accuracy must keep the difference between measured and estimated lower than the threshold. The accurate model or prediction tool can thus increase the correct fault or fault-free detection rates.

Different parameter settings will result in various FCD values, so three appropriate parameters including embedding dimension, time delay parameter and length scale should be selected seriously. Usually, two factors should be taken into consideration when setting parameters. Under faulty conditions, the FCD values should distinctly represent the characteristic of measured and estimated curve variations. On the contrary, under fault-free conditions, the FCD values characterizing measured curve variations should be identical or very similar to that for estimated curve variations.

Further investigation should be carried out on the hybrid fault detection technique combining FCD with direct residual. The novel FCD-based approach can be used to discover the small bias fault even with some noise, but it requires a period of time to observe the curve variations. As a complement, the direct residual method can detect real-time large bias faults, although its correct fault detection rate is a little low in detecting relatively small bias fault. The hybrid method will be promising to identify fault generated under noise conditions.

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